NONSTEADY HEAT EXCHANGE DURING THE STEADY MOVEMENT OF POWER-LAW FLUIDS IN THE INITIAL THERMAL SECTIONS OF A CHANNEL AND A TUBE

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The nonsteady heat exchange during the movement of power-law fluids in the initial thermal sections of a channel and a tube is studied.

Non-Newtonian media are widely used at present in the chemical industry, oil and gas production, and other branches of industry.

The problem of the nonisothermal movement of these media is of both theoretical and practical interest. It should be noted that processes of steady heat exchange are examined for the most part [1-3]. However, the problems of regulation and control of processes and apparatus which operate with high thermal loads are connected with the study of convective heat exchange in a nonsteady mode. Various cases, including variation in the wall temperature with time, can give rise to a nonsteady mode. Therefore, the study of the effects of these variations on the heat exchange during the movement of different media is of practical interest in many engineering problems.

This problem with the movement of a viscous fluid was formulated and solved in [4, 5].

The heat exchange in a steady laminar mode of movement of a power-law fluid in the thermal initial sections of a round cylindrical tube and a flat channel is examined in the present report.

It is assumed that at the initial time the temperature field in the stream of medium is uniform, i.e., the temperature of the medium at entrance is equal to the wall temperature, and, consequently, isothermal movement of the medium occurs. Starting with a certain time the wall temperature abruptly changes and assumes a new constant value T_w.

It should be noted that the solution for a single temperature jump can be generalized, as was shown in [6], to the case of arbitrary time variation in the wall temperature.

In this case the following assumptions are made: the movement proceeds in the direction of the z axis; internal heat sources are absent from the stream; the amount of heat released through energy dissipation is negligibly small; the medium is incompressible and its physicomechanical properties are constant.

1. First let us consider the problem of heat exchange in a round tube.

With allowance for what has been said above, the energy equation for a nonsteady temperature field has the form

$$\frac{\partial T}{\partial t} + v_z \frac{\partial T}{\partial z} = a \frac{1}{r} \cdot \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right). \tag{1}$$

Through the introduction of the dimensionless variables

$$\rho = \frac{r}{R}; \quad \Theta = \frac{T - T_0}{T_w - T_0}; \quad \mathrm{Pe} = \frac{\bar{v}2R}{a}; \quad Z = \frac{2}{R\,\mathrm{Pe}} z; \quad \mathrm{Fo} = \frac{at}{R^2}$$

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Eq. (1) is written in the form

$$\frac{\partial \Theta}{\partial \mathrm{Fo}} + \bar{v}_z \frac{\partial \Theta}{\partial Z} = \frac{1}{\rho} \cdot \frac{\partial}{\partial \rho}\left(\rho \frac{\partial \Theta}{\partial \rho}\right). \quad (2)$$

To solve the problem we will use the approximate method of boundary-layer theory in conjunction with the method of characteristics curves. As shown in [6], for the particular case of the given problem, i.e., when n = 1 (a viscous medium), the disagreement between the results of a calculation of the heat exchange by the method presented above and the results obtained through direct numerical integration does not exceed 5%. In this case the stream in the tube can be divided into two regions: a thermal boundary layer of thickness δ, which depends on Fo, Z, and n, and a core in which the temperature is constant and equal to the temperature T_0 at the entrance, i.e., Θ = 0. By changing from the variable ρ to the variable Y = 1 − ρ and integrating (2) over Y from 0 to δ, we obtain

$$\frac{\partial}{\partial \mathrm{Fo}} \int_0^\delta (1-Y)\Theta\, dY + \frac{\partial}{\partial Z} \int_0^\delta (1-Y)\bar{v}_z \Theta\, dY = -\left[\frac{\partial \Theta}{\partial Y}\right]_{Y=0}. \quad (3)$$

The velocity distribution in dimensionless form has the appearance

$$\bar{v}_z = \frac{v_z}{\bar{v}} = \frac{3n+1}{n+1}\left[1-(1-Y)^{\frac{n+1}{n}}\right]. \quad (4)$$

To solve Eq. (3) we assign the temperature distribution over the thickness of the boundary layer in the form of a polynomial:

$$\Theta = b_0 + b_1 Y + b_2 Y^2 + b_3 Y^3.$$

Determining the coefficients from the boundary conditions

$$\mathrm{Fo} > 0,\ Y = 0\ \Theta = 1,\ \frac{\partial}{\partial Y}\left[(1-Y)\frac{\partial \Theta}{\partial Y}\right] = 0,$$

$$\mathrm{Fo} > 0,\ Y = \delta\ \Theta = 0,\ \frac{\partial \Theta}{\partial Y} = 0,$$

we obtain as a result

$$\Theta = 1 - \frac{6}{4\delta + \delta^2}\left(Y + \frac{1}{2}Y^2 - \frac{1+\delta}{3\delta^2}Y^3\right). \quad (5)$$

Substituting (4) and (5) into (3) and performing the integration, we have

$$\frac{\partial J_1}{\partial \mathrm{Fo}} + \frac{\partial J_2}{\partial Z} = \frac{6}{\delta^2 + 4\delta}, \quad (6)$$

where

$$J_1 = \delta - \frac{\delta^2}{2} - \frac{1}{\delta+4}\left(\frac{5}{2}\delta - \frac{11}{10}\delta^2 - \frac{7}{20}\delta^3\right),$$

$$J_2 = \frac{3n+1}{n+1}\left\{J_1 + \frac{n}{3n+1}(1-\delta)^{\frac{3n+1}{n}} - \frac{n}{3n+1} - \frac{6}{4\delta+\delta^2} \times \right.$$

$$\times\left[\frac{3n^3}{(3n+1)(4n+1)(5n+1)} + \frac{2n(38n^3+12n+1)}{3(4n+1)(5n+1)(6n+1)}\delta + \right.$$

$$+ \frac{n(8n+1)}{6(5n+1)(6n+1)}\delta^2 - \frac{2n^3}{(4n+1)(5n+1)(6n+1)}\frac{1}{\delta} -$$

$$\left. - \frac{2n^4}{(3n+1)(4n+1)(5n+1)(6n+1)}\frac{1+\delta}{\delta^2}\right](1-\delta)^{\frac{3n+1}{n}} +$$

$$\left. + \frac{6}{4\delta+\delta^2}\left[\frac{n^2(6n+1)}{(3n+1)(4n+1)(5n+1)} - \frac{2n^4}{(3n+1)(4n+1)(5n+1)(6n+1)} \cdot \frac{1+\delta}{3\delta^2}\right]\right\}.$$

Integrating (6) by the method of characteristics curves, we obtain

$$\mathrm{Fo} = f(\delta) \tag{7}$$

$$Z = \frac{3n+1}{n+1}\Bigg\{ f(\delta) + \frac{n^3(4248n^4+4332n^3+1591n^2+204n+9)}{3(3n+1)^2(4n+1)^2(5n+1)^2(6n+1)} - \\ - \Bigg[\frac{n^3(4248n^4+4332n^3+1591n^2+204n+9}{3(3n+1)^2(4n+1)^2(5n+1)^2(6n+1)} - \\ - \frac{n^4(144n+30)}{3(4n+1)^2(5n+1)^2(6n+1)}\delta - \frac{6n^3}{6(5n+1)^2(6n+1)}\delta^2 \Bigg](1-\delta)^{\frac{3n+1}{n}} - \\ - \frac{n^2[8(6n+1)^2+3n^2]}{8(3n+1)(4n+1)(5n+1)(6n+1)} \ln\left(\frac{\delta}{4}+1\right) + \\ + \frac{n^4}{(3n+1)(4n+1)(5n+1)(6n+1)}[\varphi_1(\delta)+\varphi_2(\delta)+\varphi_3(\delta)]\Bigg]\Bigg\}, \tag{8}$$

where

$$f(\delta) = -\frac{13}{15}\delta + \frac{7}{30}\delta^2 - \frac{1}{36}\delta^3 - \frac{1}{80}\delta^4 + \frac{52}{15}\ln\left(\frac{\delta}{4}+1\right),$$

$$\varphi_1(\delta) = \int_0^\delta \left[\frac{10n^2-7n-2}{n^2}\cdot\frac{1}{\delta} - \frac{2(8n+3)}{n}\cdot\frac{1}{\delta^2} - \frac{6}{\delta^3}\right](1-\delta)^{\frac{2n+1}{n}}d\delta,$$

$$\varphi_2(\delta) = -\int_0^\delta \left[\frac{142n+63}{n} + \frac{2(3n+1)}{n}\cdot\frac{1}{\delta} + \frac{2}{\delta^2}\right]\frac{(1-\delta)^{\frac{2n+1}{n}}}{\delta+4}d\delta,$$

$$\varphi_3(\delta) = \int_0^\delta \left[\frac{6}{\delta^3} + \frac{4,5}{\delta^2} - \frac{8(6n+1)^2-3n^2}{8n^2}\cdot\frac{1}{\delta}\right]d\delta.$$

Equations (7) and (8) are valid in the Z–Fo plane along the characteristic curves in one case when δ depends only on Fo and in another case when δ depends only on Z. The two equations merge along the limiting characteristic curve which passes through the origin of coordinates and divides the Z–Fo plane into two regions. The region lying between the limiting characteristic curve and the Z axis corresponds to the nonsteady mode of heat exchange, while the region lying between the limiting characteristic curve and the Fo axis corresponds to the steady mode. Consequently, the time Fo_s required for the attainment of a steady mode of heat exchange with any value of Z is determined by the equation of the limiting characteristic curve. The heat flux density at the wall is found from Eq. (5):

$$\frac{q_W R}{(T_W - T_0)\lambda} = \frac{6}{4\delta-\delta^2}. \tag{9}$$

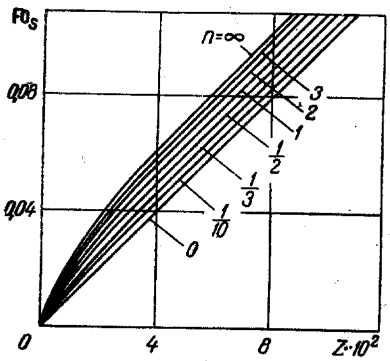
Calculations from Eqs. (7), (8), and (9) are presented in Figs. 1, 2, 3, and 4 for a Newtonian liquid (n = 1), a dilatant liquid (n > 1), and a pseudoplastic liquid (n < 1), and graphs are also presented for extreme pseudoplastic (n = 0) and extreme dilatant (n = ∞) materials. All the quantities in the figures are given in dimensionless form.

It is seen that with a decrease in the rheological parameter n the time required for the establishment of the temperature field and the heat flux at the wall decreases and with an increase in n it increases, with this time changing insignificantly when n > 3.

The above is obviously connected with the fact that ever fuller velocity profiles occur for pseudoplastic media.

The time of establishment of the temperature field and the heat flux at the wall for different n strongly depends on the length Z of the initial thermal section. Other conditions being equal, this dependence increases with an increase in Z.

It should be noted that in the particular case when n = 1 (a viscous liquid) the results fully coincide with the data obtained in [6].



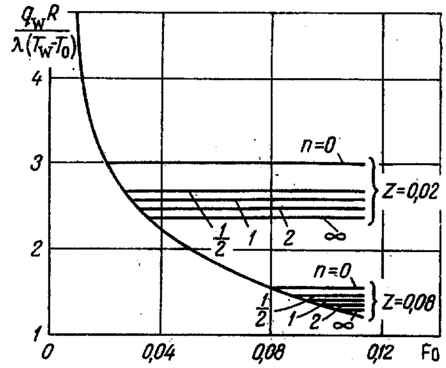
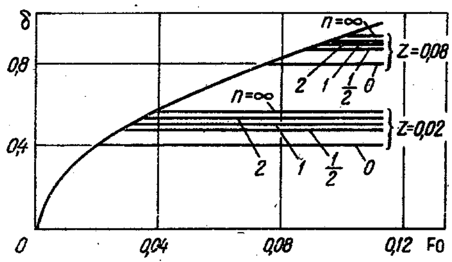


Fig. 1 Fig. 2

Fig. 1. Time of onset of the steady mode in the thermal initial section of a round tube for different n upon a sudden change in the wall temperature.

Fig. 2. Variation in the thickness of the boundary layer as a function of the time and length of the initial section for different n.



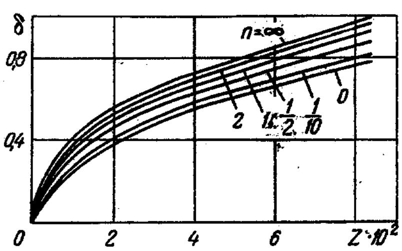


Fig. 3 Fig. 4

Fig. 3. Variation in the thickness of the boundary layer as a function of the length of the initial section for different n.

Fig. 4. Variation in the heat flux density at the wall as a function of the time and the length of the initial thermal section for different n.

Example 1. Transformer oil ($n = 1$, $a = 2.04 \cdot 10^{-4}$ m²/h [6]) or polyethylene ($n = 1/3$, $a = 4.7 \cdot 10^{-4}$ m²/h [2, 3]) is flowing with Pe = 2000 in a tube with diameter d = 20 under isothermal conditions. At a certain time the wall temperature abruptly changes. To be determined: the time following the jump in which a steady state sets in at a distance z = 40d from the entrance. In this case $Z = (2/\mathrm{Pe}) \cdot (z/R) = 0.08$. This value, as seen from Fig. 1, corresponds to $\mathrm{Fo}_s = 0.096$ for transformer oil and $\mathrm{Fo}_s = 0.085$ for polyethylene, where $\mathrm{Fo}_s = at_s/R^2$.

Hence we find that for transformer oil the steady mode sets in after $t_s = 170$ sec and for polyethylene after $t_s = 65$ sec.

Example 2. A viscous medium ($n = 1$, $a = 2 \cdot 10^{-4}$ m²/h) or a power-law medium with the parameters $n = 1/3$ or $n = 3$ ($a = 2 \cdot 10^{-4}$ m²/h) is flowing with Pe = 2000 in a tube with diameter d = 20 mm under isothermal conditions. At a certain time the wall temperature abruptly changes. To be determined: the time following the jump in which a steady state sets in at a distance z = 40d from the entrance. In this case Z = 0.08. This value, as seen from Fig. 1, corresponds to $\mathrm{Fo}_s = 0.096$ for the viscous medium, $\mathrm{Fo}_s = 0.085$ for $n = 1/3$, and $\mathrm{Fo}_s = 0.103$ for $n = 3$, where $\mathrm{Fo}_s = at_s/R^2$. Hence we find that $t_s = 173$ sec for the viscous medium, $t_s = 151$ sec for $n = 1/3$, and $t_s = 187$ sec for $n = 3$.

2. The nonsteady heat exchange during the steady laminar movement of a power-law fluid in the thermal initial section of a flat channel is examined.

The energy equation for a nonsteady temperature field in dimensionless form has the following appearance:

$$\frac{\partial\Theta}{\partial\,\mathrm{Fo}}+\frac{2n+1}{n+1}\left[1-(1-Y)^{\frac{n+1}{n}}\right]\frac{\partial\Theta}{\partial Z}=\frac{\partial^2\Theta}{\delta Y^2},$$

$$\Theta=\frac{T-T_0}{T_c-T_0},\quad Y=\frac{y}{h};\ \mathrm{Fo}=\frac{a}{h^2}t,\quad Z=\frac{2}{h\,\mathrm{Pe}}z,\ \mathrm{Pe}=\bar{v}\,\frac{2h}{a}. \tag{10}$$

Using the approximate boundary-layer method applied above in conjunction with the method of characteristic curves, we obtain the following expression for the temperature field:

$$\Theta=1-\frac{3}{2}\left(\frac{Y}{\delta}\right)+\frac{1}{2}\left(\frac{Y}{\delta}\right)^3. \tag{11}$$

Here the dependence of the thickness δ of the thermal boundary layer on Z and Fo has the form

$$\mathrm{Fo}=(8\delta)^{\frac{1}{2}},$$

$$Z=\frac{2n+1}{n+1}\left\{\frac{1}{8}\delta^2+\frac{2n^3}{(5n+1)(6n+1)}\delta(1-\delta)^{\frac{2n+1}{n}}-\right.$$

$$\left.-\frac{2n^3(13n^3+28n^2+14n+2)}{(2n+1)^2(3n+1)^2(4n+1)(5n+1)}\left[(1-\delta)^{\frac{2n+1}{n}}-1\right]-\varphi_1(\delta)-\varphi_2(\delta)\right\}, \tag{12}$$

$$\varphi_1(\delta)=\frac{2n^4}{(2n+1)(3n+1)(4n+1)(5n+1)}\int\limits_0^\delta\left[\frac{3}{\delta^3}+\frac{3(n+1)}{n}\,\frac{1}{\delta^2}-\frac{7n^2-1}{n^2}\cdot\frac{1}{\delta}\right](1-\delta)^{\frac{n+1}{n}}d\delta,$$

$$\varphi_2(\delta)=\frac{n^2}{(2n+1)(3n+1)}\int\limits_0^\delta\left[\frac{1}{\delta}-\frac{6n^2}{(4n+1)(5n+1)}\cdot\frac{1}{\delta^3}\right]d\delta. \tag{13}$$

The heat flux density at the wall is found from Eq. (11):

$$\frac{q_c 2h}{(T_c-T_0)\lambda}=\frac{3}{\delta}.$$

As is seen, the expressions obtained have no qualitative differences from the expressions obtained in the preceding section. Consequently, the conclusions drawn in Sec. 1 do not lose meaning for Sec. 2.

NOTATION

T, temperature of moving medium; T_0, temperature at entrance to tube; T_w, temperature of tube wall; v_z, steady velocity of medium; $\bar{v}$, averaged (over the cross section) velocity; $\bar{v}_z$, dimensionless velocity; b_0, b_1, b_2, b_3, coefficients; ρ, Z, dimensionless coordinates; t, time; R, tube radius; a, coefficient of thermal diffusivity; Θ, dimensionless temperature; Pe and Fo, Peclet and Fourier numbers, respectively; n, exponent of nonlinearity of fluid; $Y = 1 - \rho$, new dimensionless variable; δ, thermal boundary layer; q_w, heat flux density at wall; 2h, distance between plates of channel; λ, coefficient of thermal conductivity; t_s, time of onset of steady mode.

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